

Exploration of the mathematics of String Art and its applications with GeoGebra

Joachim König
Korea Nat'l Univ. of Education
(joint with Dongwan Kim, Jinwook Park)

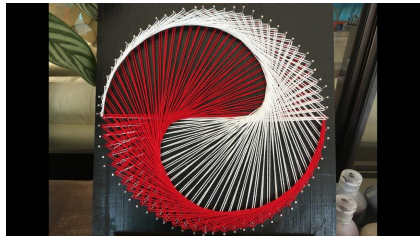
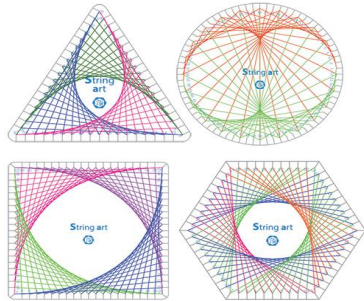
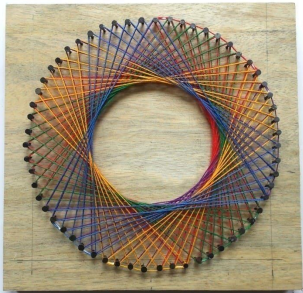
KNUE-Unitwin Conference
Nov. 3rd 2023

Motivation: Role of Math Education in the digital era

- **Increased** importance of mathematics, and thus of math education, in the advent of the digital age.
- At the same time, **change of paradigm**.
 - What should math do for us?
 - What should we know about math?
 - How should we be doing math?
- From “static” (computation, memorization) to “dynamic” (experimentation, changing and evolving scenarios,...)
- Increased **interconnectivity** of subjects in STEM/STEAM education.

String Arts

- Introduction



String Art

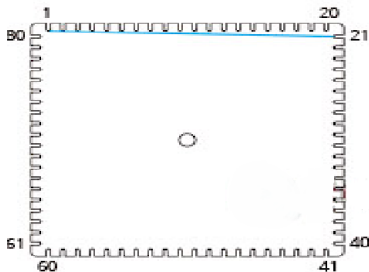


Objectives

- Combined **Hands-on** and **Digital** activity
- **Experimentation** and **conjecture building** via **dynamical** functionality of computer program
- **Mathematical modelling** and **visualization** of (simple) “real-life” problems
- **Application** of familiar contents (recognition of familiar objects in an a priori unfamiliar context)
- **Connection** of aspects from geometry/algebra/calculus.
- **Aesthetic** dimension of math → motivation via **enjoyment**.

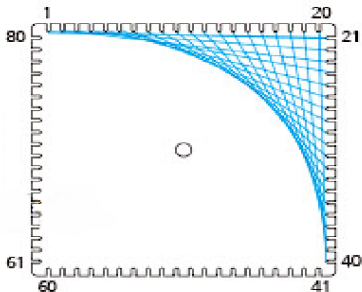
String Arts by hand

- Step 1: Put string to number 1 and number 21



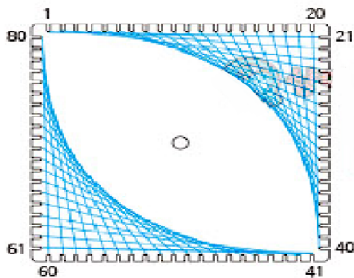
String Arts by hand

- Step 2: Second string to number 2 and 22.
- More generally, string to number $(k, 20+k)$ (k from 1 to 20)



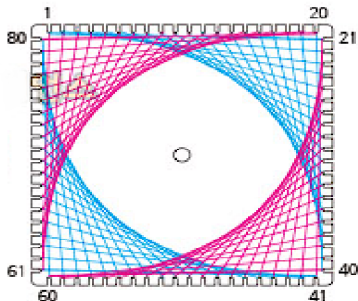
String Arts by hand

- Step 3: Repeat on other side of design



String Arts by hand

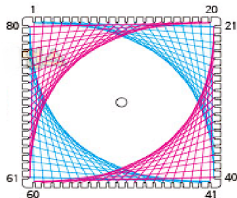
- Step 4: Repeat it with other color string



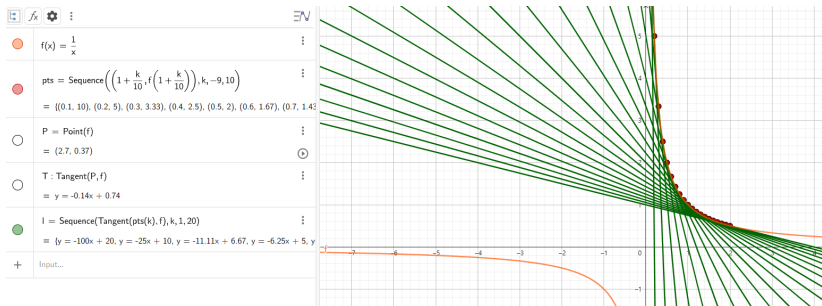
Mathematics of String Art

Questions:

- 1) What kind of curve do we see arising?
- 2) How does the curve change if we change to rule for the strings?



Sample: Obtaining a prescribed curve as envelope

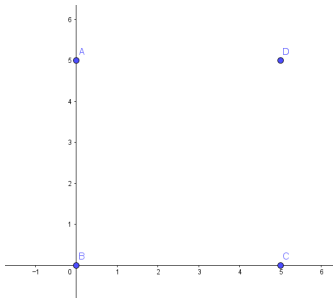


Sample: Obtaining a prescribed curve as envelope



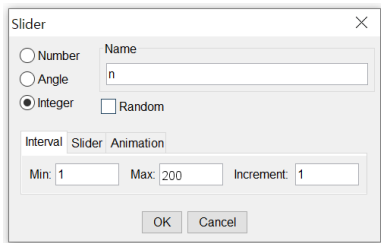
2nd Lesson: String Arts using GeoGebra

- Step 1: Create Reference Points



String Arts using GeoGebra

- Step 2: Build Integer Slider



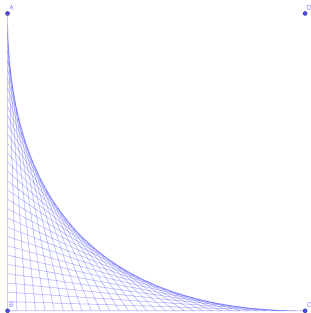
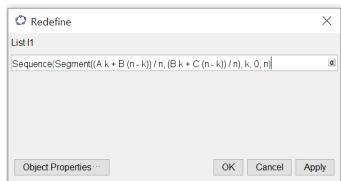
The image shows the 'Slider' dialog box in GeoGebra. It has a title bar with a close button (X). The dialog is divided into several sections:

- Type Selection:** Three radio buttons are present: 'Number', 'Angle', and 'Integer'. The 'Integer' option is selected.
- Name:** A text input field contains the letter 'n'.
- Random:** A checkbox labeled 'Random' is currently unchecked.
- Interval:** A tabbed section with three tabs: 'Interval', 'Slider', and 'Animation'. The 'Interval' tab is active, showing three input fields: 'Min: 1', 'Max: 200', and 'Increment: 1'.
- Buttons:** 'OK' and 'Cancel' buttons are located at the bottom of the dialog.



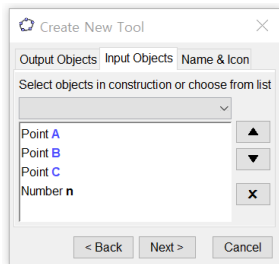
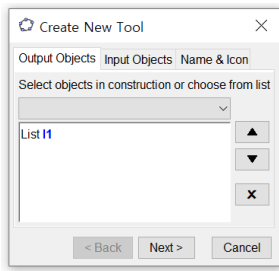
String Arts using GeoGebra

- Step 3: Build String Art Segments using Sequence



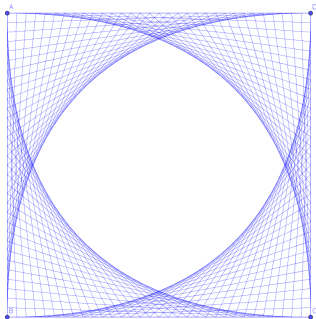
String Arts using GeoGebra

- Step 4: Create a New Tool



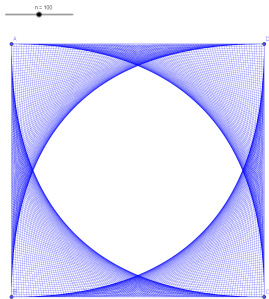
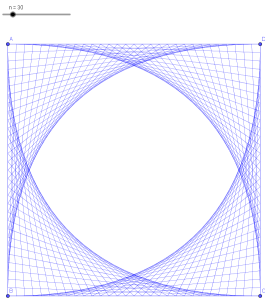
String Arts using GeoGebra

- Step 5: Create All String Art Segments



String Arts using GeoGebra

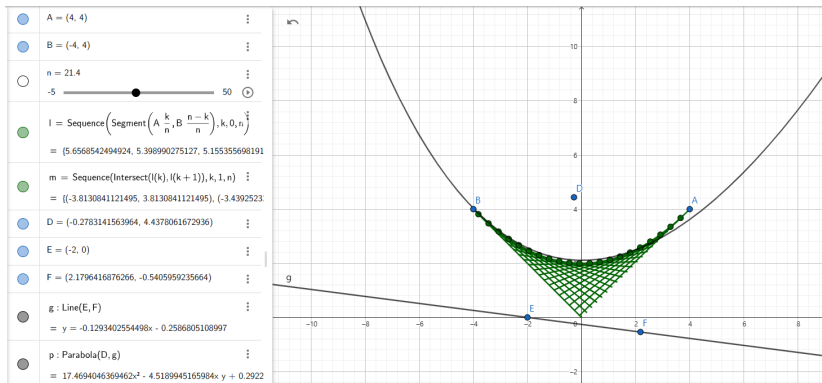
- Step 6: Change the Number of Strings




Sample: Experimental determination of an enveloping curve equation

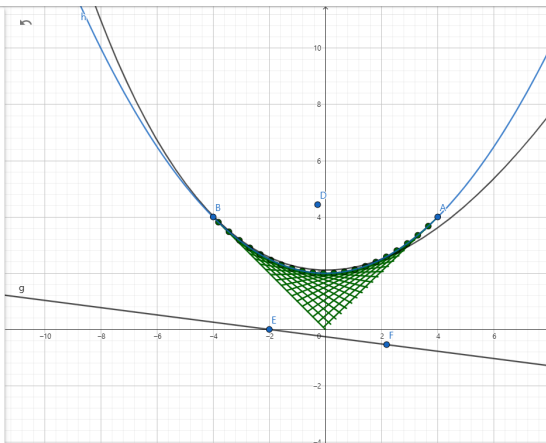
- 1) **Experimentation**, by dragging a parabola to fit the prescribed shape.
- 2) **Concrete equation**, by computer algebra.
- 3) **Verification of correctness**, e.g. via tangent equations.

Sample: Experimental determination

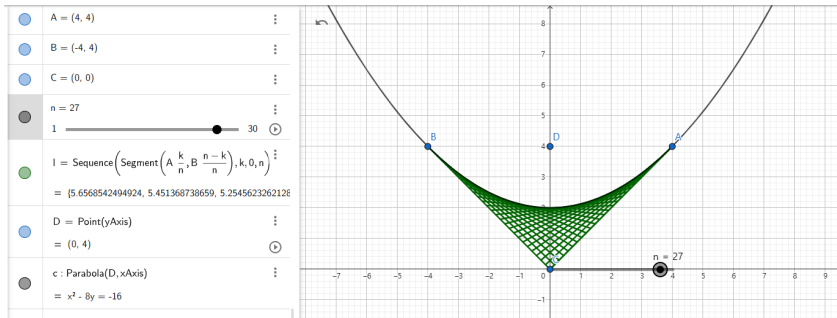


Sample: Experimental determination

| | | |
|--------------------------------------|--|---|
| ● | A = (4, 4) | ⋮ |
| ● | B = (-4, 4) | ⋮ |
| ○ | n = 21.4 | ⋮ |
| | -5  50 | ⋮ |
| ● | $l = \text{Sequence}\left(\text{Segment}\left(A \frac{k}{n}, B \frac{n-k}{n}\right), k, 0, n\right)$ = {5.6568542494924, 5.398990275127, 5.15355696191} | ⋮ |
| ● | $m = \text{Sequence}(\text{Intersect}(l(k), l(k+1)), k, 1, n)$ = {(-3.8130841121495, 3.8130841121495), (-3.4392523} | ⋮ |
| ● | D = (-0.2783141563964, 4.4378061672936) | ⋮ |
| ● | E = (-2, 0) | ⋮ |
| ● | F = (2.1796416876266, -0.5405959235664) | ⋮ |
| ● | $g : \text{Line}(E, F)$ = $y = -0.1293402554498x - 0.2586805108997$ | ⋮ |
| ● | $p : \text{Parabola}(D, g)$ = $17.4694046369462x^2 - 4.5189945165984x + 0.2922$ | ⋮ |
| ● | $h(x) = \text{FitPoly}(m, 2)$ = $0.125x^2 + 0x + 1.9956328063586$ | ⋮ |

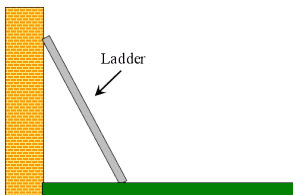


Example: String Art and Parabolas

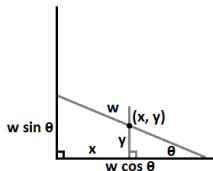


Hidden string art in everyday problems

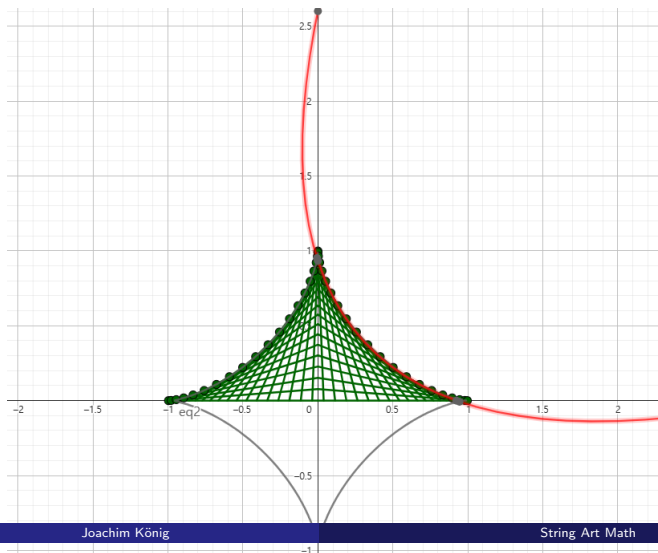
- Example: Sliding ladder.
- A ladder of length L is leaned vertically against a wall.
- Now the ladder begins to slide down, until it lies horizontally on the ground.
- Question: What kind of curve arises as the ladder is moving?



Modelling the problem with string art

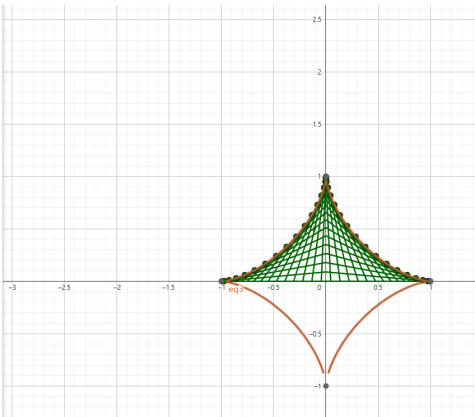


Sliding ladder problem: Experimental determination of enveloping curve

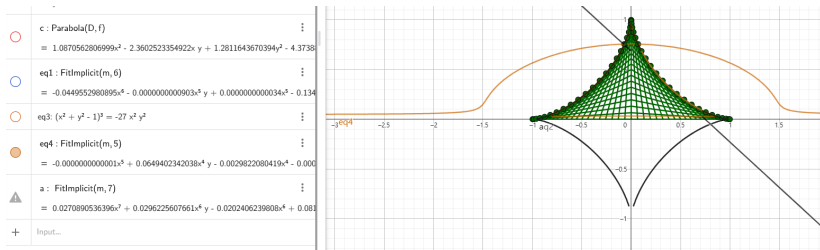


Sliding ladder problem: Experimental determination of enveloping curve

| | |
|---|--|
| ● | <code>m = Sequence(Intersect(l((k), l((k + 1))), k, 1, 2 n)</code> <code>= {(-0.9959742939952, 0), (-0.972078446573, 0.0021506651944), (-0.92562</code> |
| ○ | <code>ladder = Segment(A cos($\frac{\pi n}{2 \cdot 500}$), B sin($\frac{\pi n}{2 \cdot 500}$))</code> <code>= 1</code> |
| ● | <code>eq2: (x² + y² - 1)² = -27 x² y²</code> |
| ○ | <code>D = (0.7979586405738, 0.8407696141733)</code> |
| ○ | <code>E = (-0.8123892370596, 0.6301032621104)</code> |
| ○ | <code>F = (0.230230674845, -0.5017820532107)</code> |
| ○ | <code>f : Line(E, F)</code> <code>= y = -1.0856164383562x - 0.2518398479851</code> |
| ○ | <code>c : Parabola(D, f)</code> <code>= 1.0870562806999x² - 2.3602523354922x y + 1.2811643670394y² - 4.3738</code> |
| ● | <code>eq1 : FitImplicit(m, 6)</code> <code>= -0.0450827512824x⁶ + 0.00000000000391x⁵ y - 0.00000000000024x⁵ - 0.135</code> |
| ● | <code>eq3: (x² + y² - 1)² = -27 x² y²</code> |
| + | Input... |



Sliding ladder problem: Experimental determination of enveloping curve



Requirements / Opportunities (on teacher's side)

- Basic digital skills and familiarity with some basic GeoGebra commands
- Willingness to embrace students' suggestions
- Possibility for free generalization and adaptation of contents / lesson plan