Exploration of the mathematics of String Art and its applications with GeoGebra

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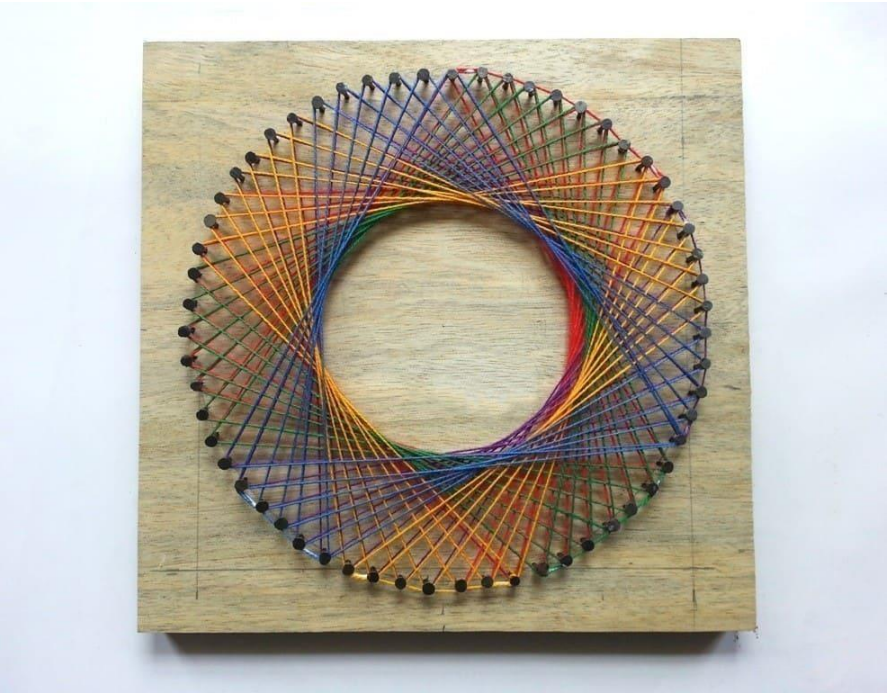
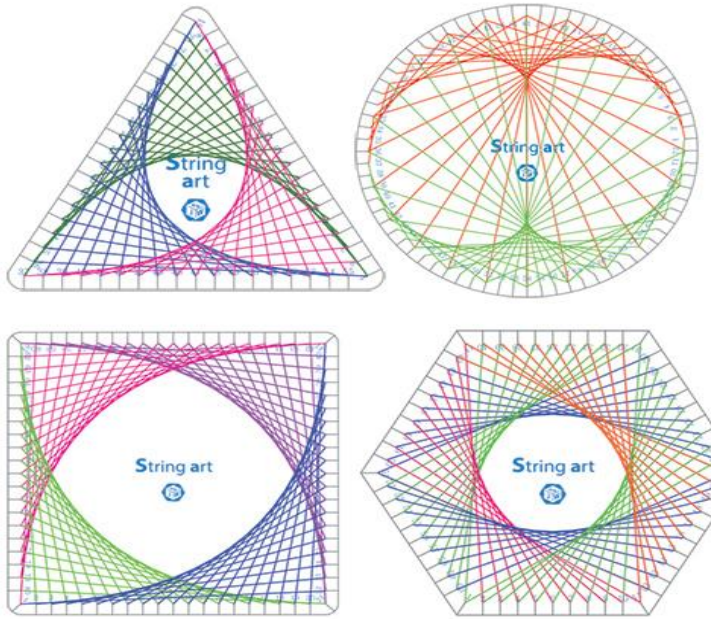
Motivation: Role of Math Education in the digital era

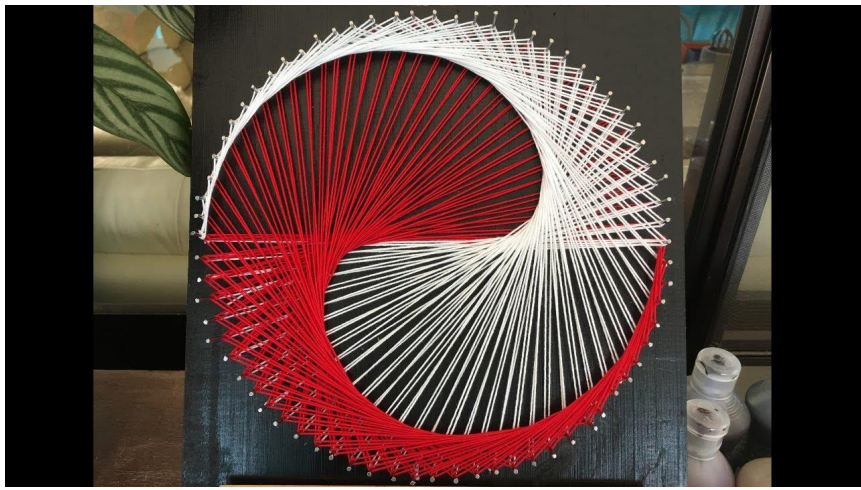
- **Increased** importance of mathematics, and thus of math education, in the advent of the digital age.
- At the same time, **change of paradigm**.
  - What should math do for us?
  - What should we know about math?
  - How should we be doing math?
- From "static" (computation, memorization) to "dynamic" (experimentation, changing and evolving scenarios,...)
- Increased **interconnectivity** of subjects in STEM/STEAM education.

String Arts

- Introduction







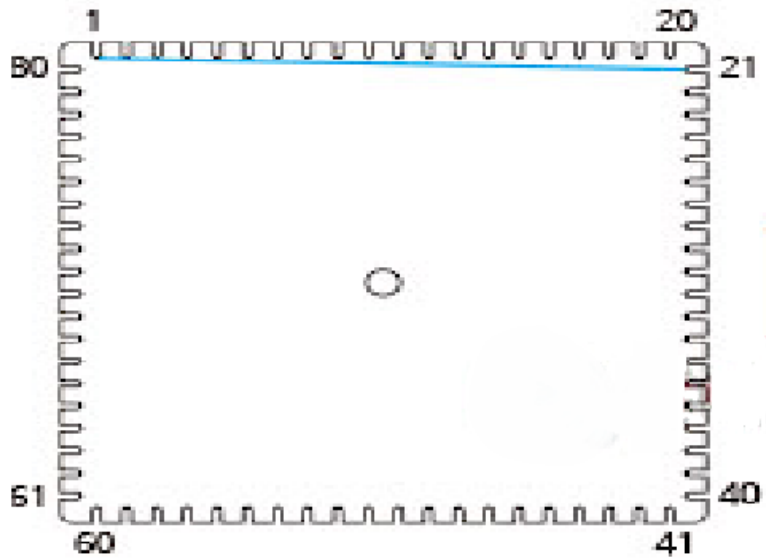
String Art

Objectives

- Combined **Hands-on** and **Digital** activity
- **Experimentation** and **conjecture building** via **dynamical** functionality of computer program
- **Mathematical modelling** and **visualization** of (simple) "real-life" problems
- **Application** of familiar contents (recognition of familiar objects in an a priori unfamiliar context)
- **Connection** of aspects from geometry/algebra/calculus.
- **Aesthetic** dimension of math $\rightarrow$ motivation via **enjoyment**.

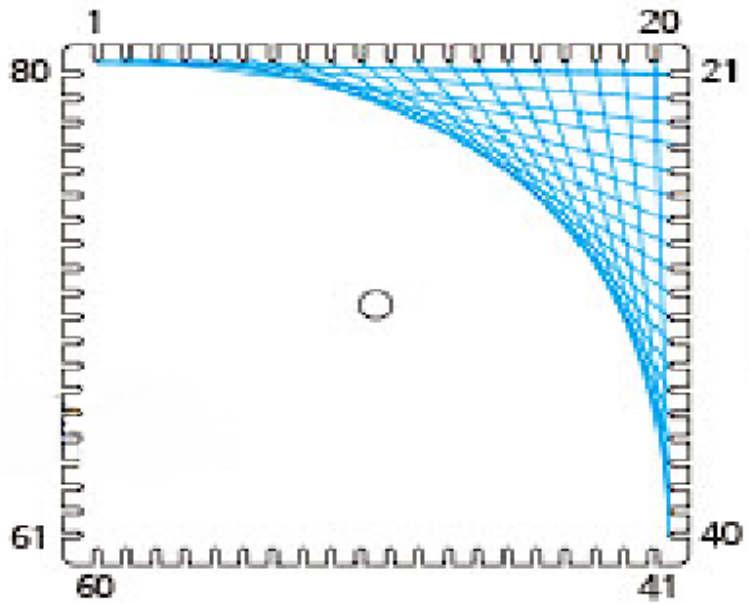
String Arts by hand

- Step 1: Put string to number 1 and number 21



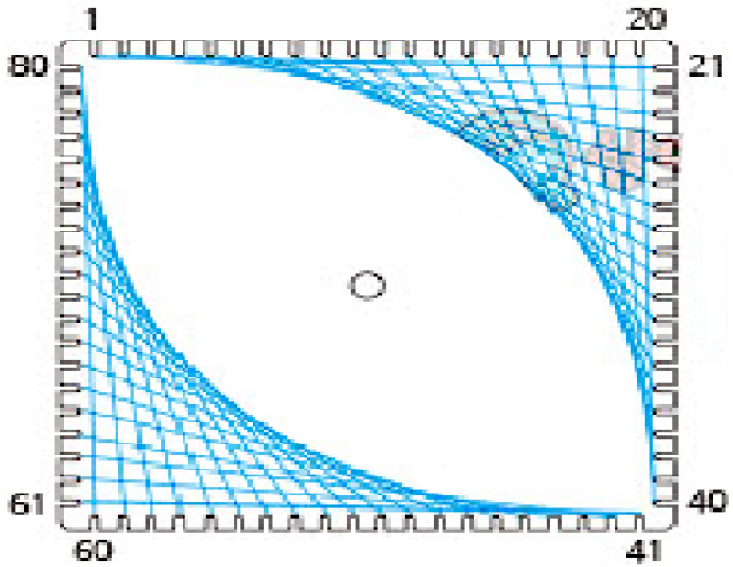
String Arts by hand

- Step 2: Second string to number 2 and 22.
- More generally, string to number (k, 20+k) (k from 1 to 20)



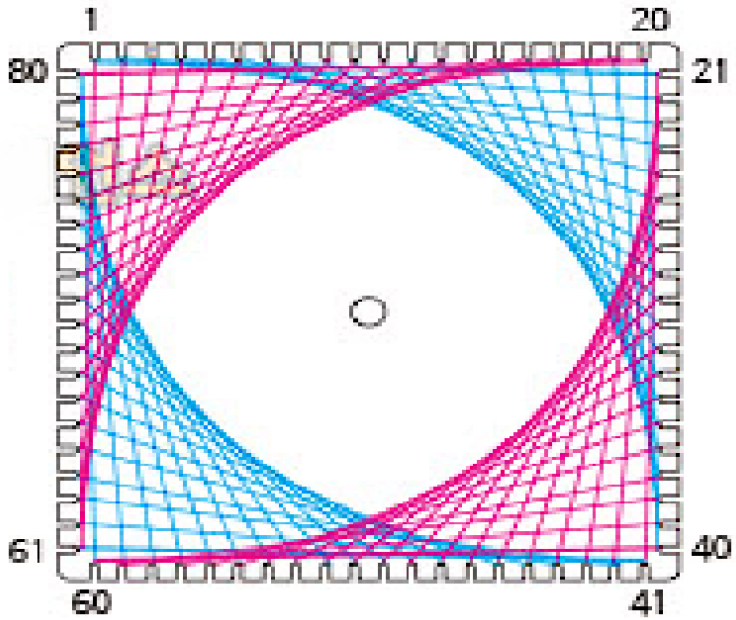
String Arts by hand

- Step 3: Repeat on other side of design



String Arts by hand

- Step 4: Repeat it with other color string

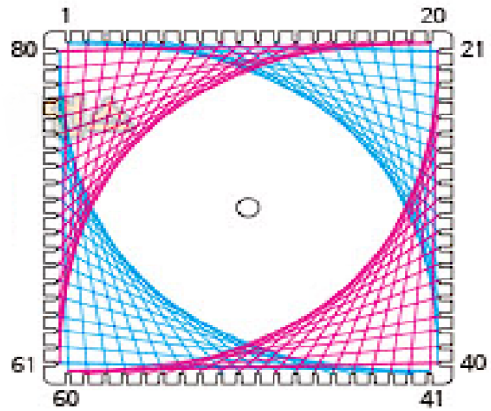


Mathematics of String Art

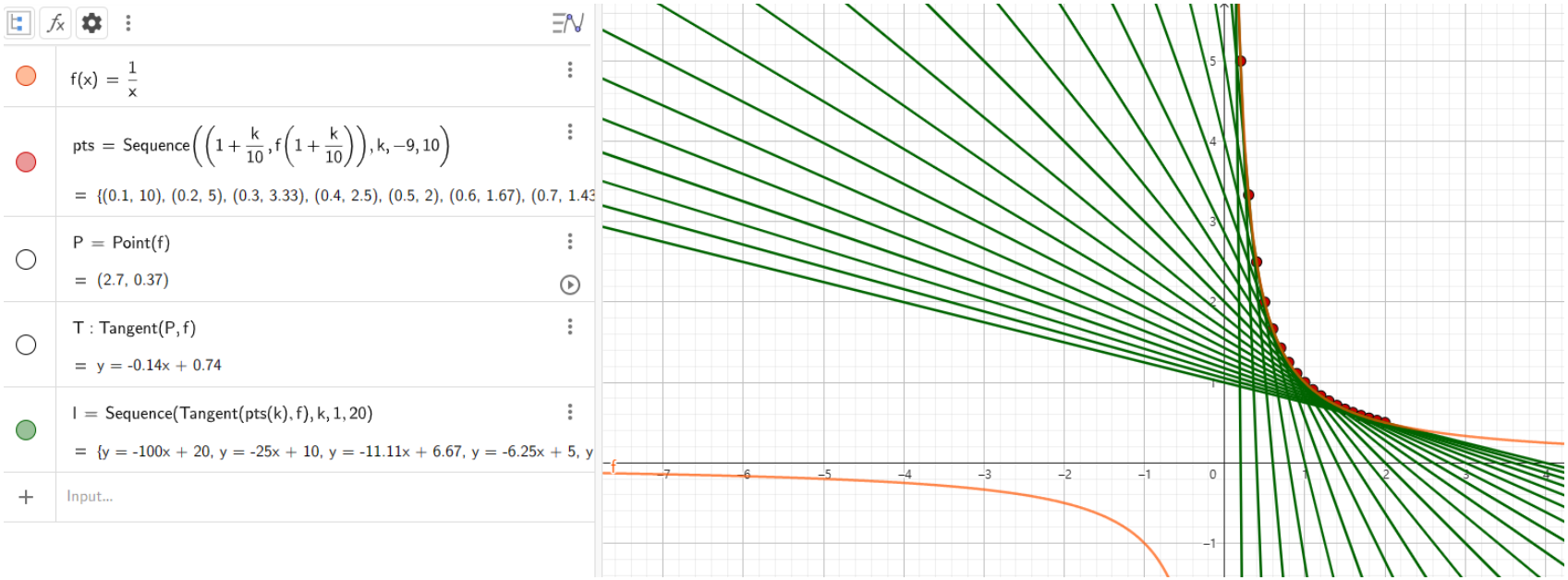
Questions:

1) What kind of curve do we see arising?

2) How does the curve change if we change to rule for the strings?



Sample: Obtaining a prescribed curve as envelope



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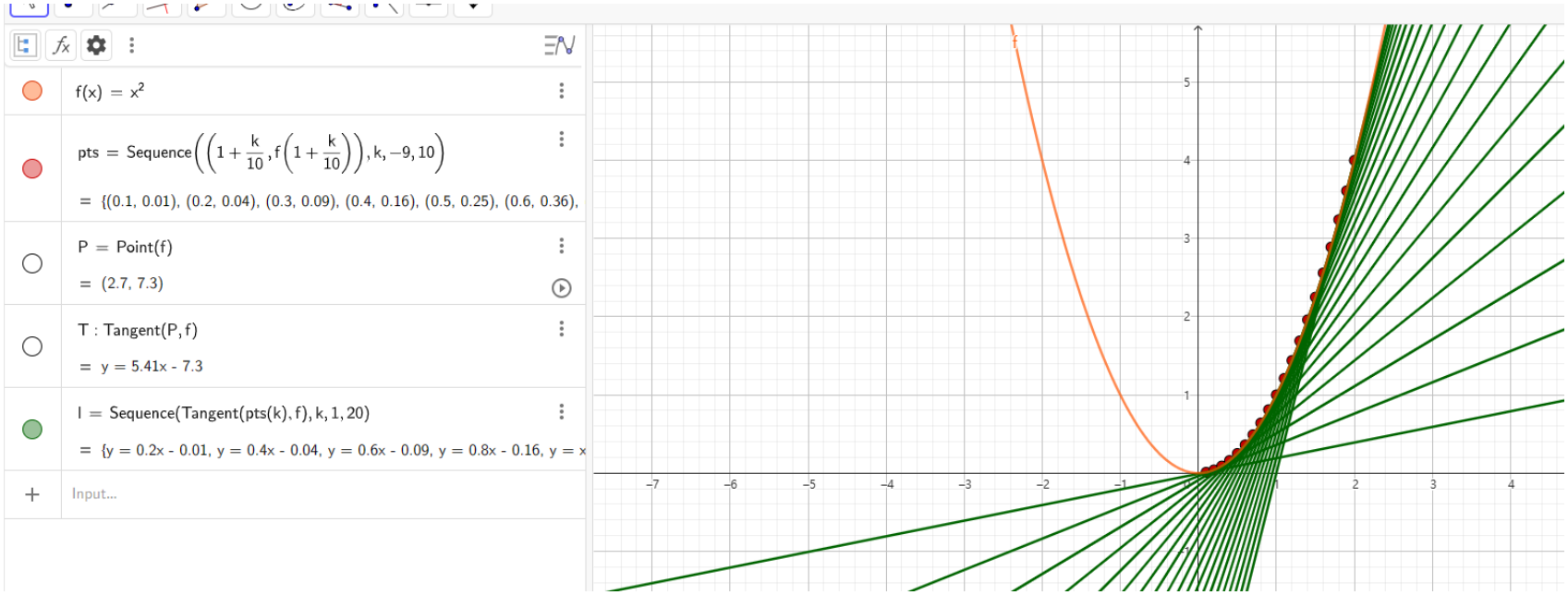
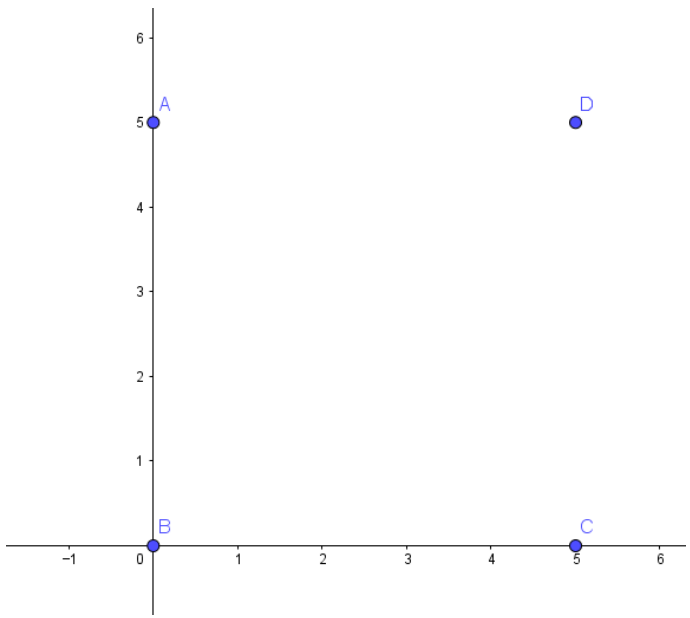


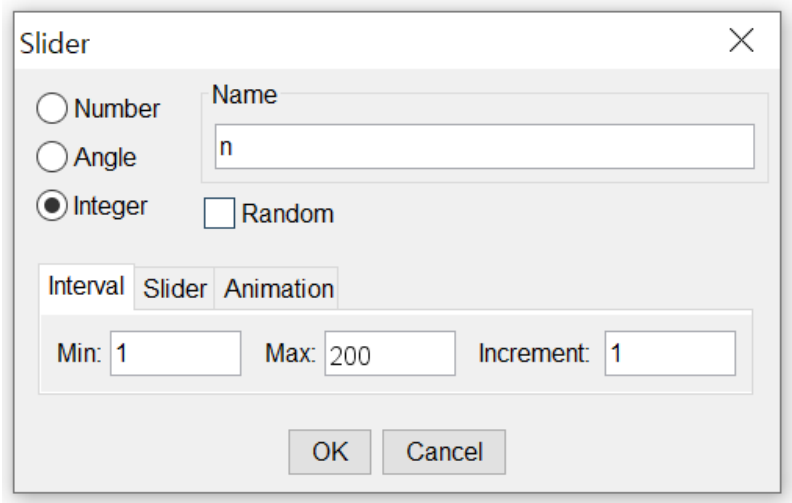
2nd Lesson: String Arts using GeoGebra

- Step 1: Create Reference Points



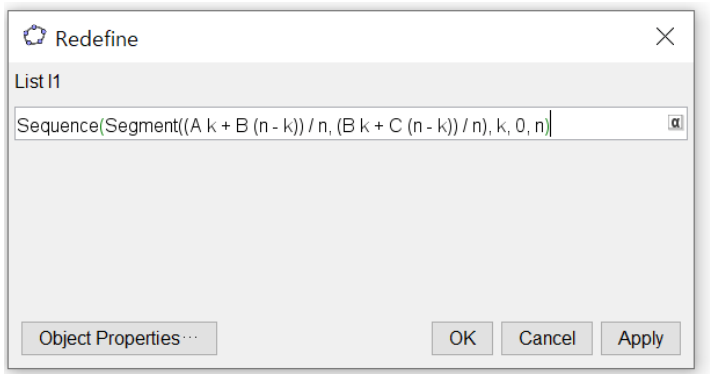
String Arts using GeoGebra

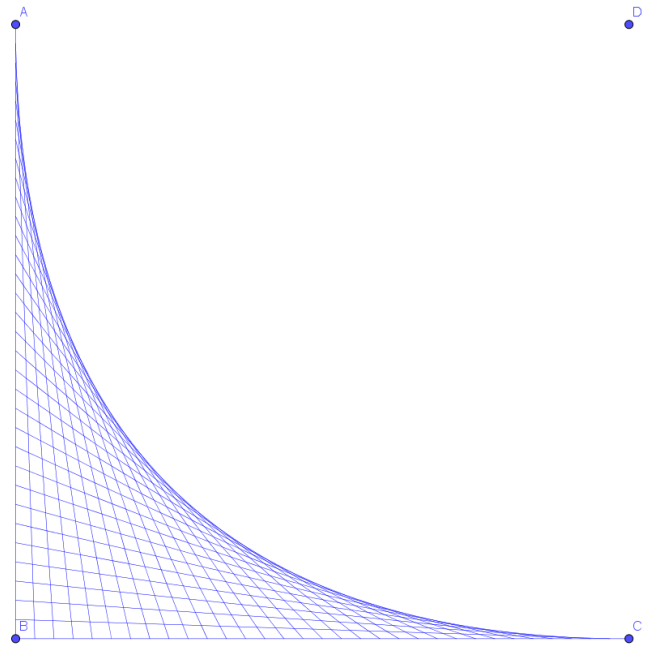
- Step 2: Build Integer Slider



String Arts using GeoGebra

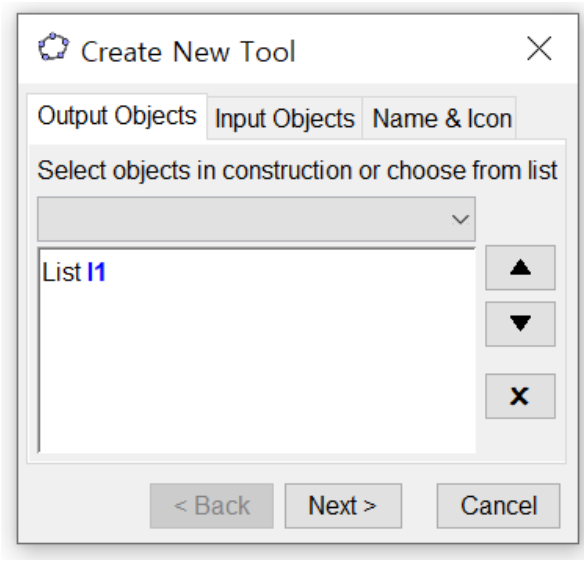
- Step 3: Build String Art Segments using Sequence

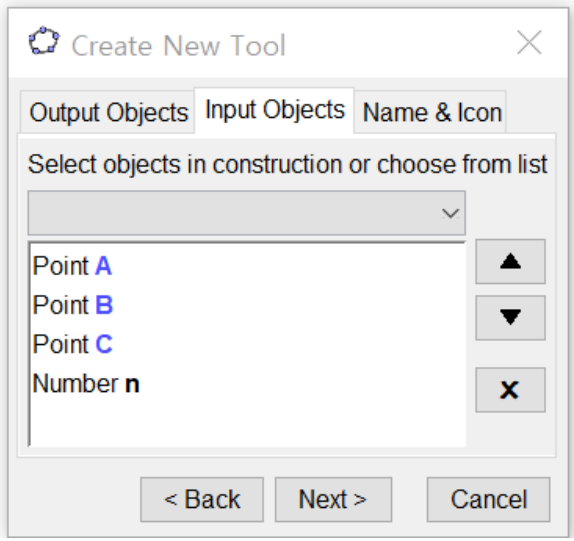




String Arts using GeoGebra

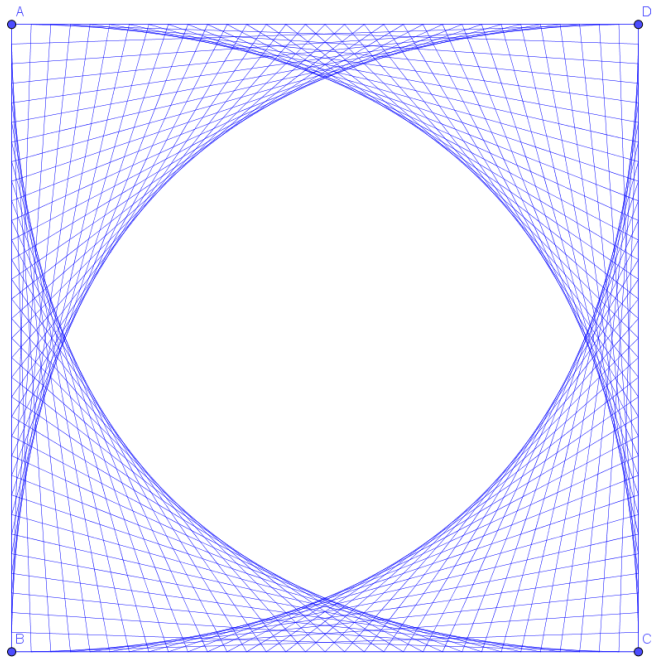
- Step 4: Create a New Tool





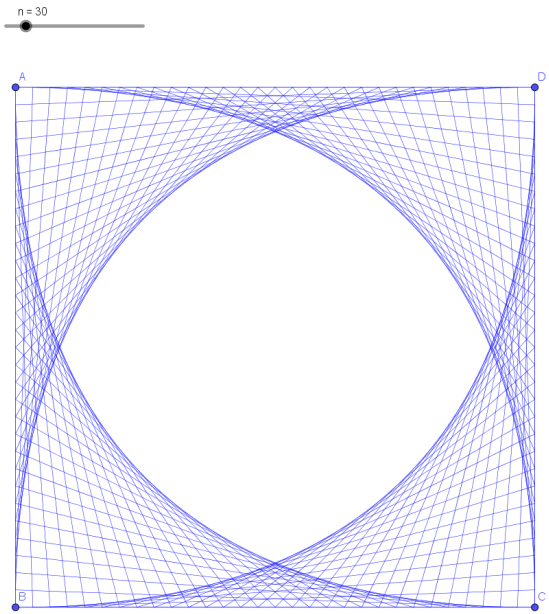
String Arts using GeoGebra

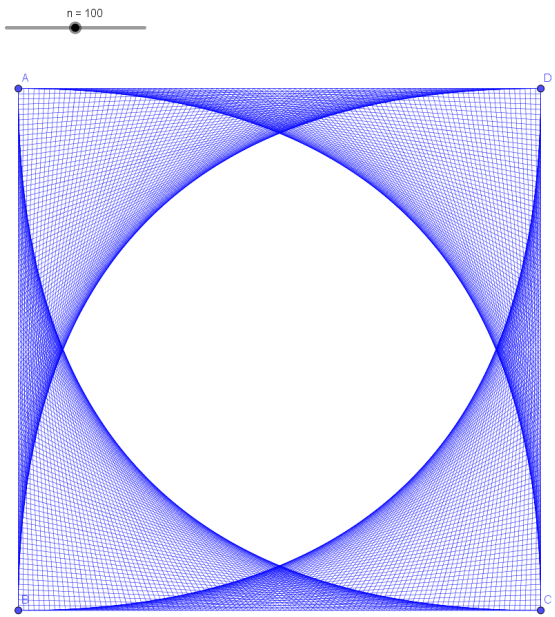
- Step 5: Create All String Art Segments



String Arts using GeoGebra

- Step 6: Change the Number of Strings

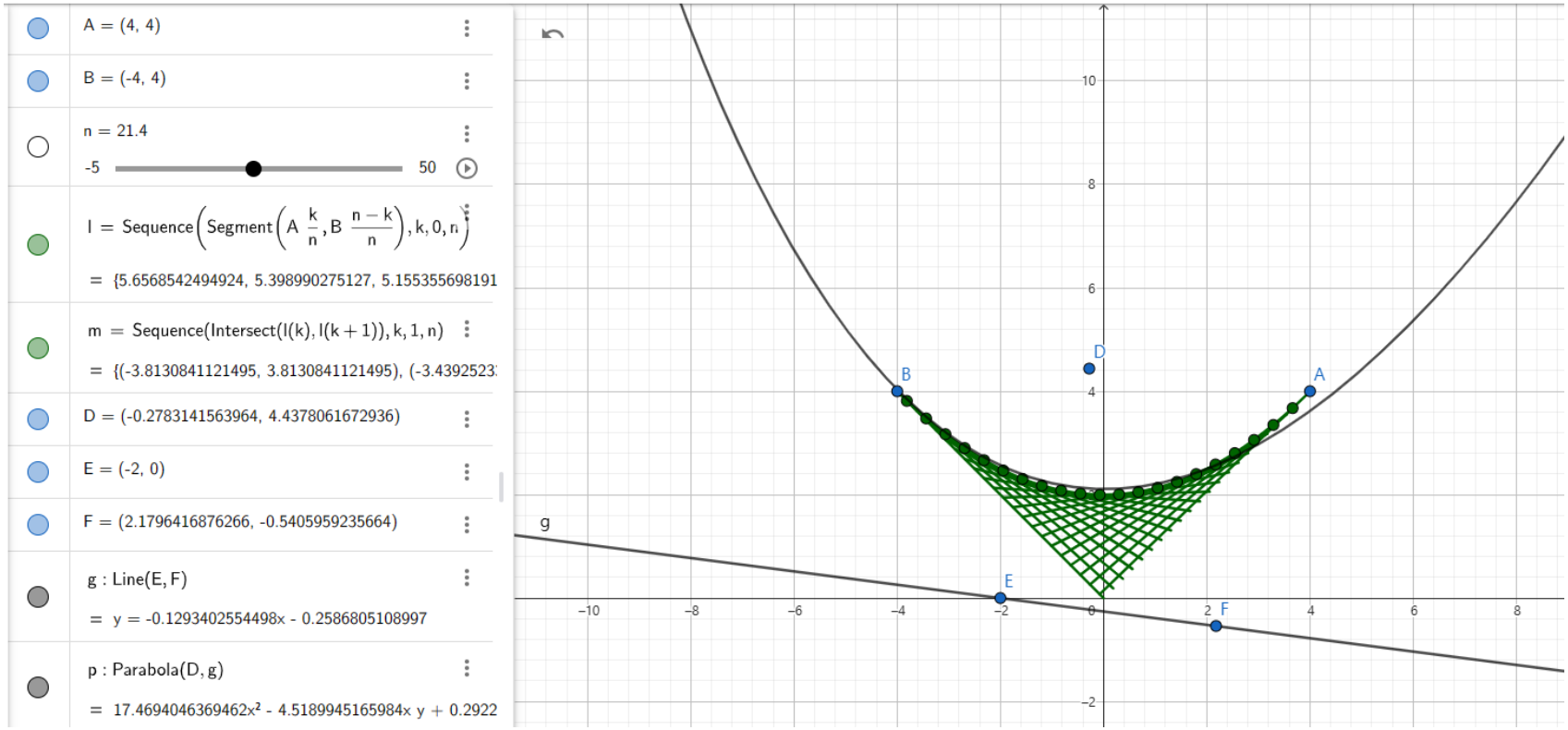




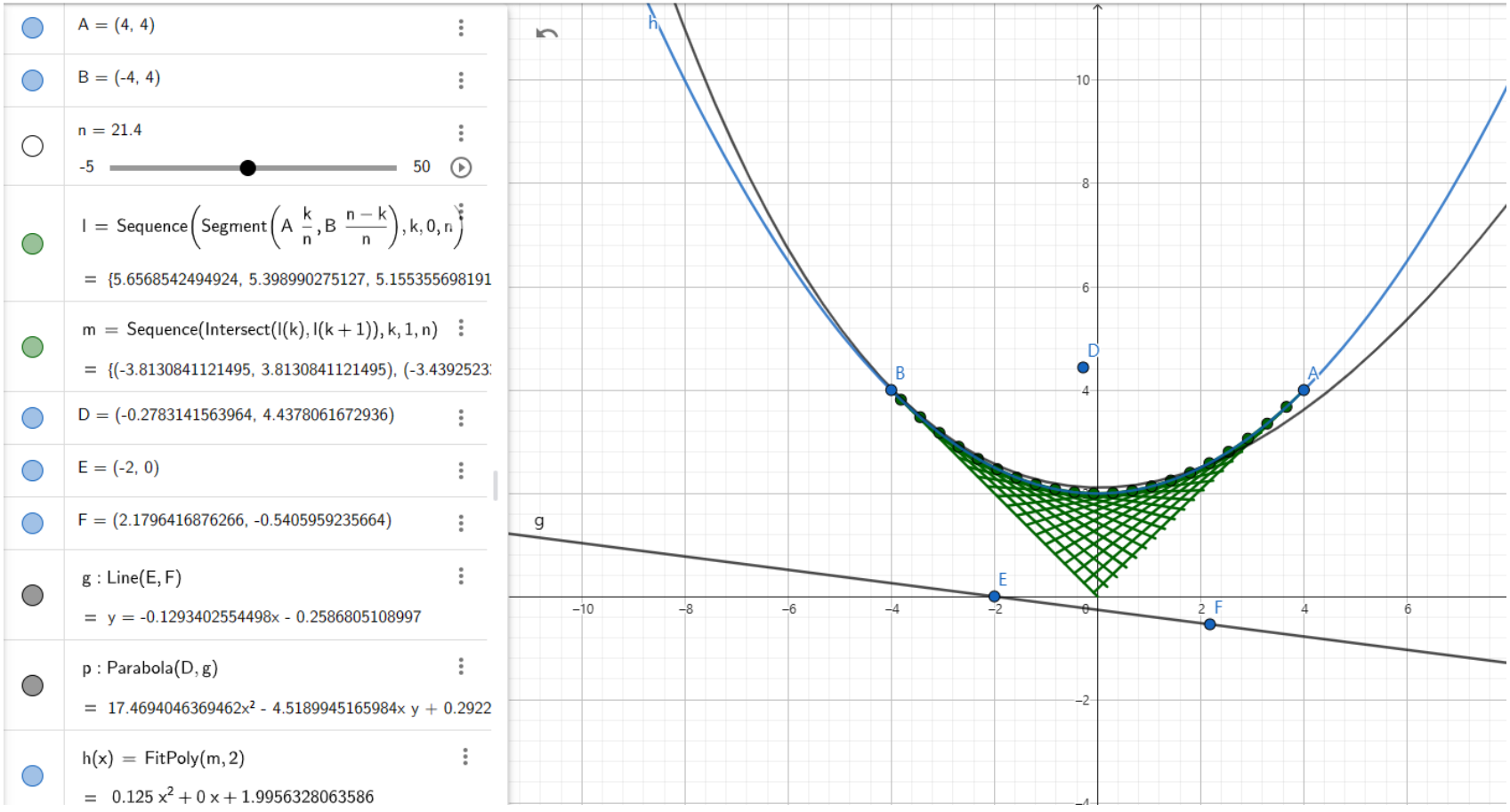
Sample: Experimental determination of an enveloping curve equation

1) **Experimentation**, by dragging a parabola to fit the prescribed shape.
2) **Concrete equation**, by computer algebra.
3) **Verification of correctness**, e.g. via tangent equations.

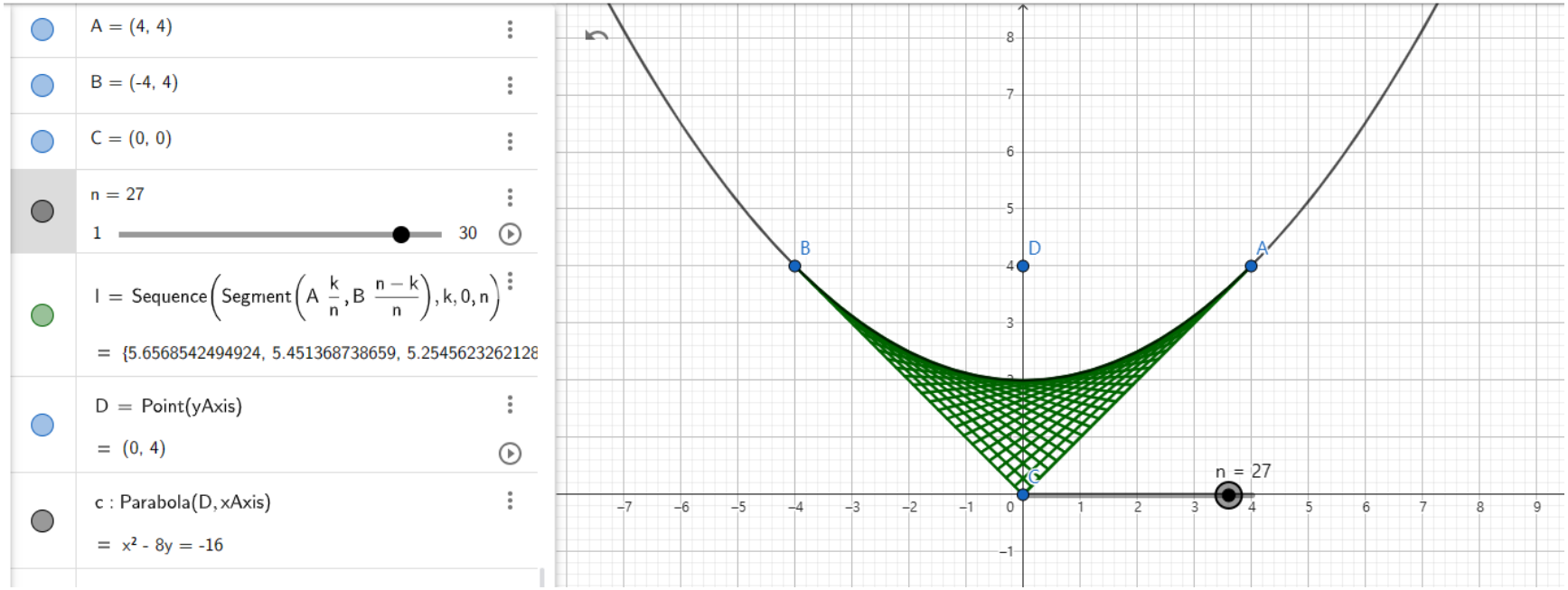
Sample: Experimental determination



Sample: Experimental determination

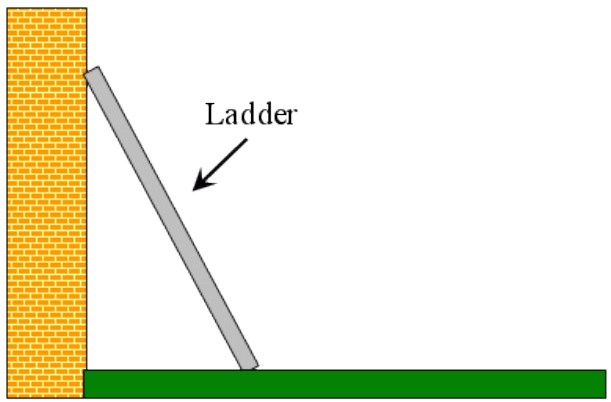


Example: String Art and Parabolas

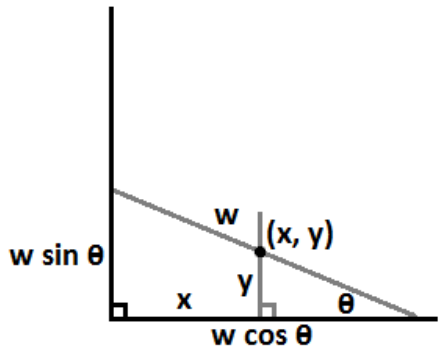


Hidden string art in everyday problems

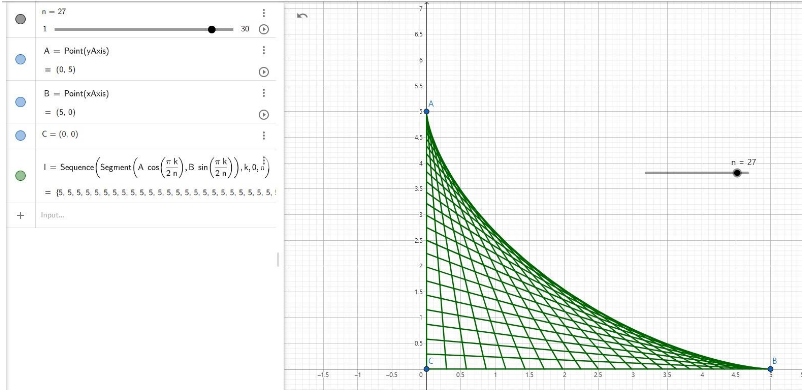
- Example: Sliding ladder.
- A ladder of length L is leaned vertically against a wall.
- Now the ladder begins to slide down, until it lies horizontally on the ground.
- Question: What kind of curve arises as the ladder is moving?



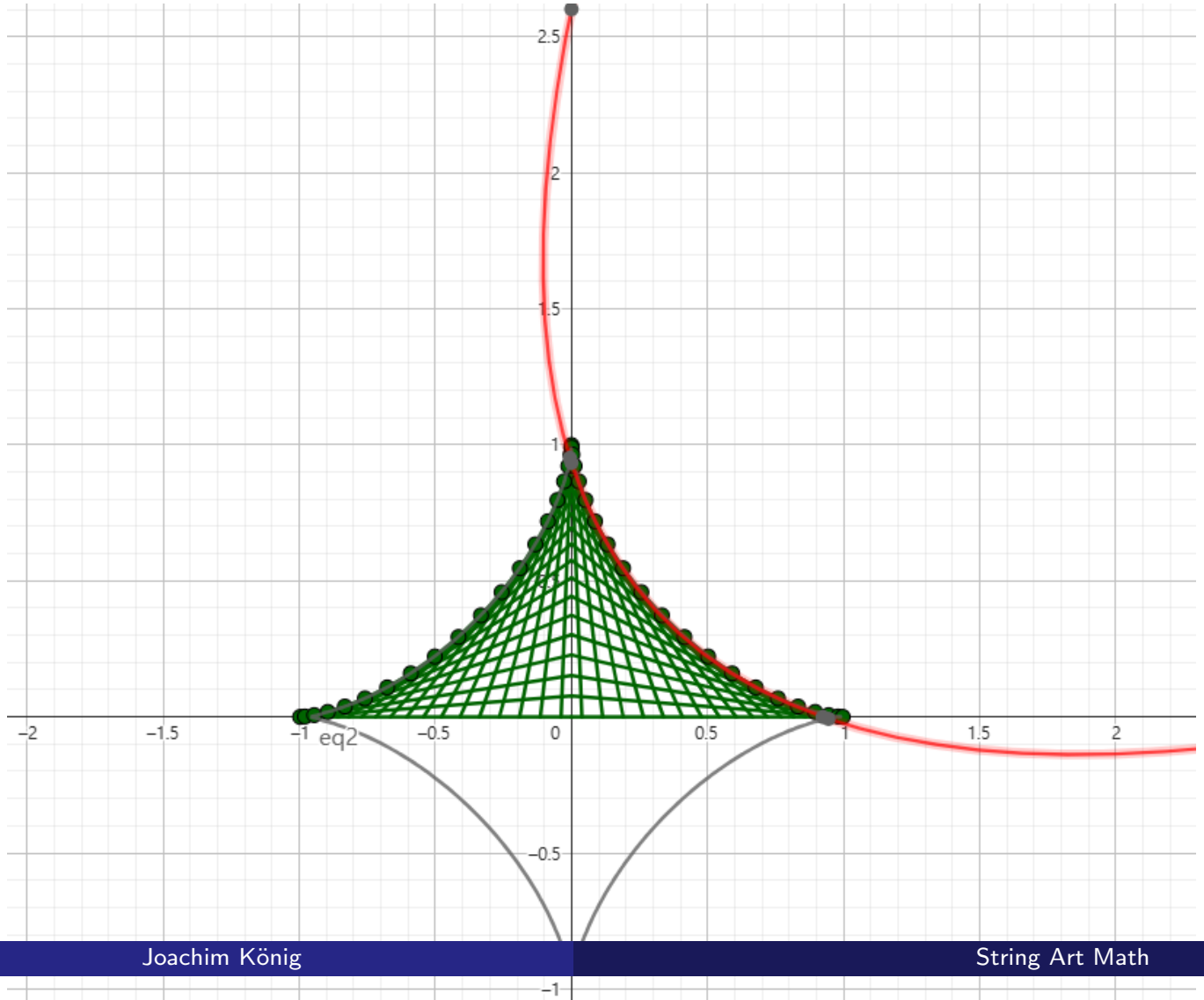
Modelling the problem with string art



Modelling the problem with string art



Sliding ladder problem: Experimental determination of envelopping curve



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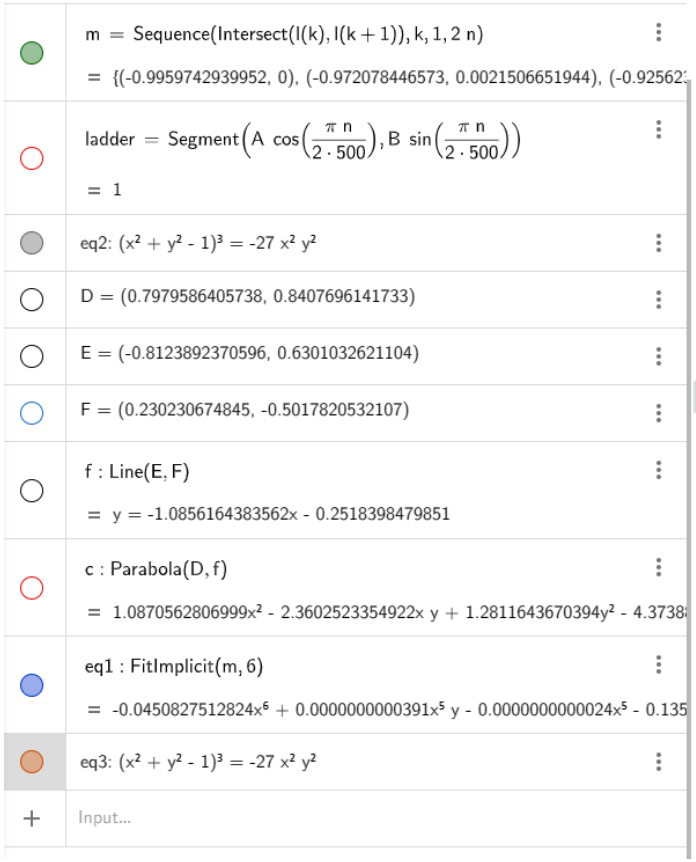

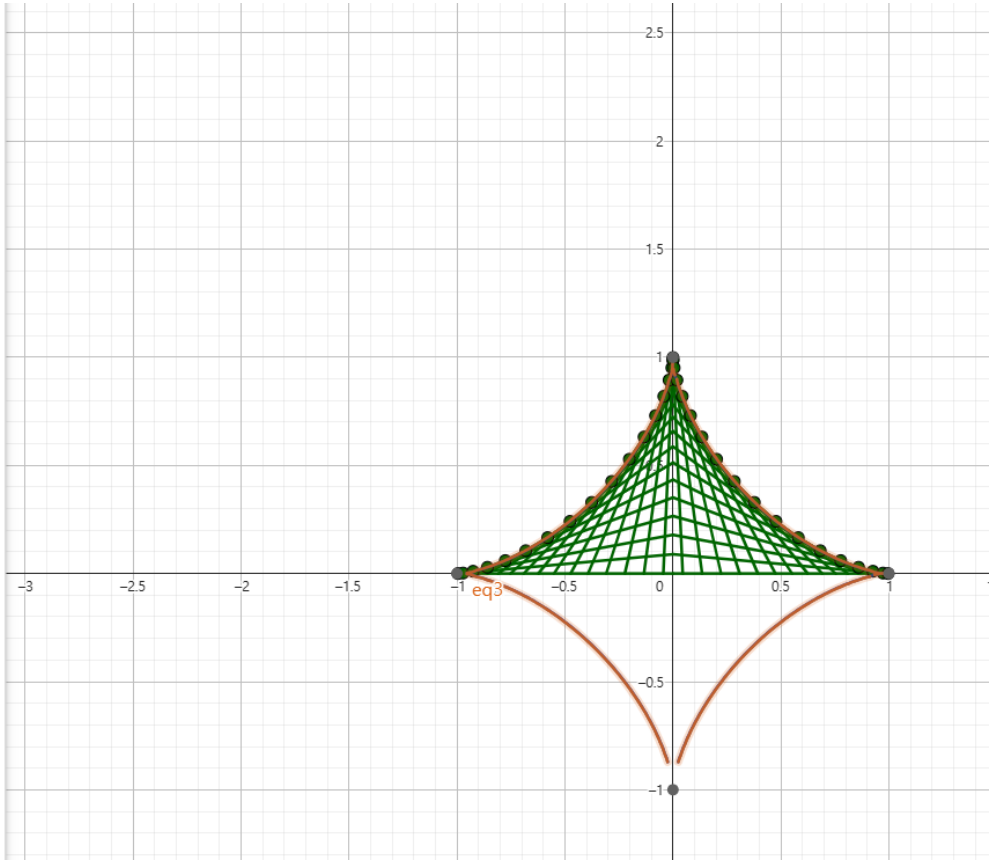

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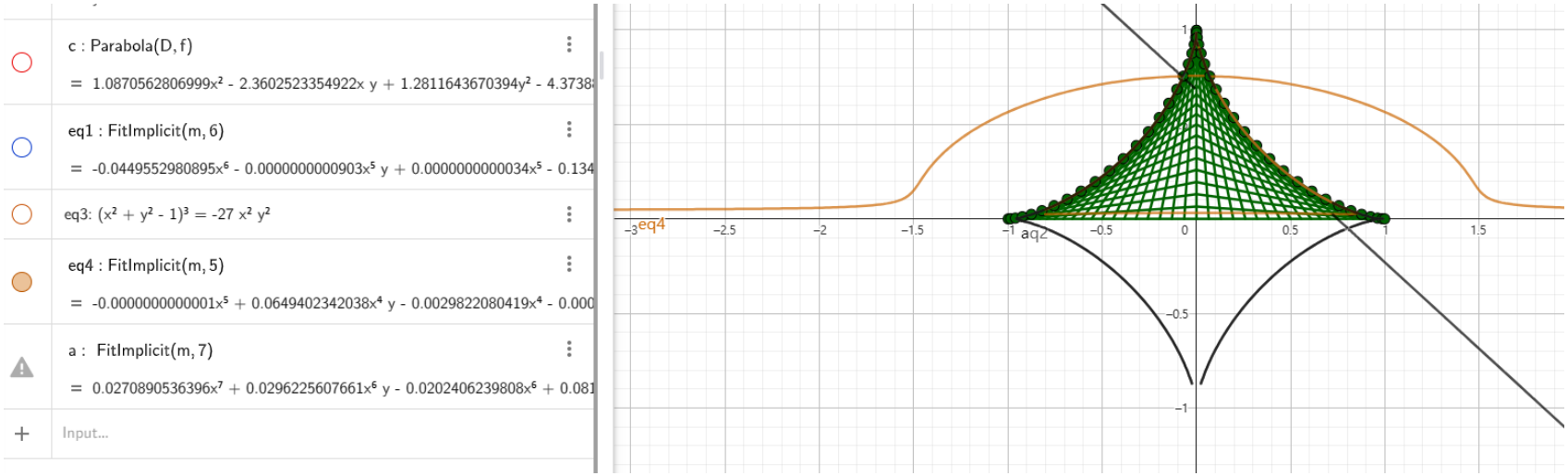

Requirements / Opportunities (on teacher's side)

- Basic digital skills and familiarity with some basic GeoGebra commands
- Willingness to embrace students' suggestions
- Possibility for free generalization and adaptation of contents / lesson plan