



UNITWIN
International Seminar

**Intuitive Model for
Teaching
Negative Numbers**

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The historico-genetic principle of Negative Numbers

1. No evidence of negative numbers in ancient Babylonia, Egypt, and Greece
2. Diophantus(AD 200 ~ 284?) : Negative numbers are 'absurd'
3. Brahmagupta(598 ~ 665): Mention the arithmetic with negative numbers
4. Bhāskara(1114 ~ 1185): The equation has two square roots of a positive number, a positive root and a negative root
5. But! European mathematicians refuse to admit negative numbers
 - Descartes(1596 - 1650): Negative solution of equation is 'wrong solution'
 - Pascal: There are not exist "less than zero"

The historico-genetic principle of Negative Numbers

Then, when could the negative numbers be accepted as numbers in mathematics?

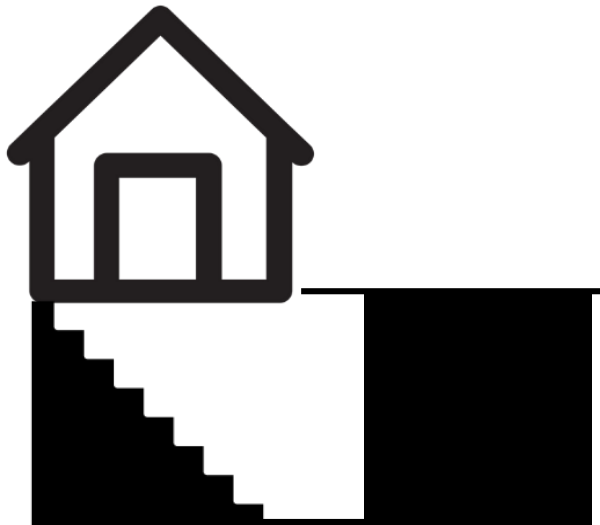
Peacock(1791 ~ 1858): Principle of the performance of equivalent forms

““Whatever algebraic forms are equivalent when the symbols are general in form, but specific in value, will be equivalent likewise when the symbols are general in value as well as in form.”

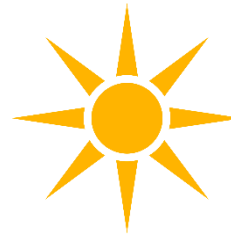
Hankel(1839 ~ 1873): The negative number is not a concept representing the actual numbers but forms a formal frame

• The historico-genetic principle of Negative Numbers

If positive represents above ground level, then negative represents below ground level.



Temperature



5°C



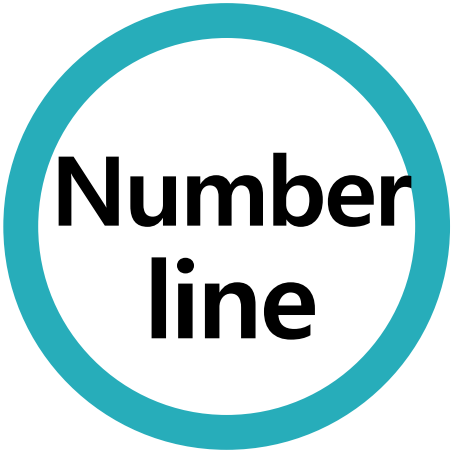
-7°C

Epistemological Obstacles on the Negative numbers

Epistemological Obstacle for learning Negative number

1. In reality, numbers less than 0 (Zero) are not suitable
2. Difficult to understand multiplication and division of negative numbers
 - $(-4) \times 3 = (-4) + (-4) + (-4) = (-12)$
 - $(4) \times (-3) = ?$
 - $(-4) \times (-3) = (12) ?$

Intuitive Models for learning Negative numbers



**Number
line**



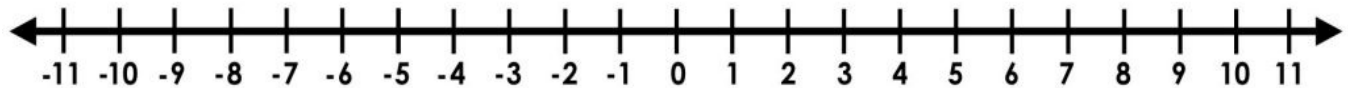
**Counti
ng**



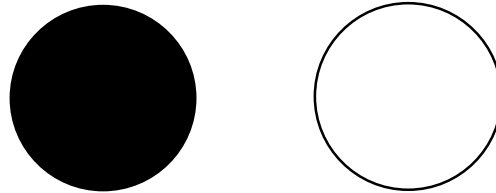
**Post
man**

Intuitive Models for learning Negative numbers

Number
line



Counting

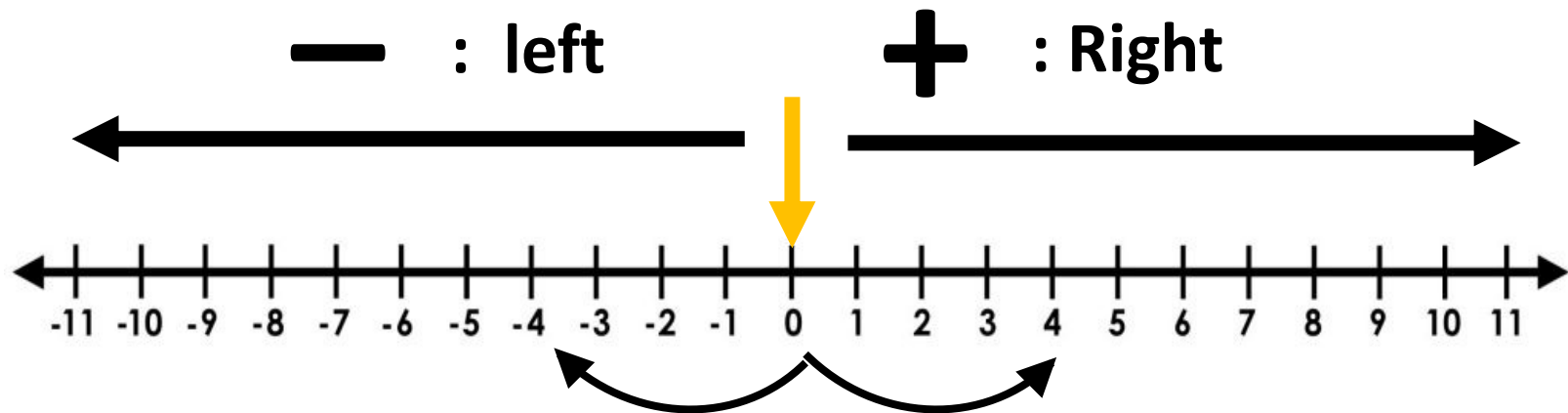


Postman

Check

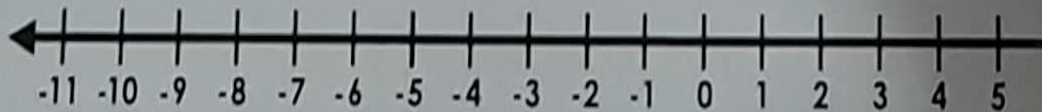
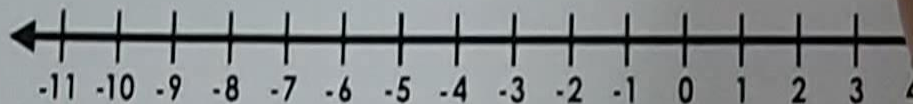
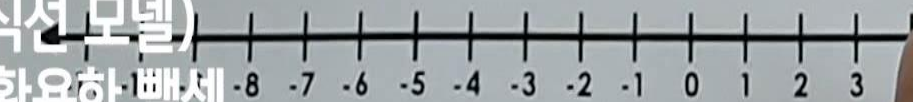
Bill

1. Number line Model

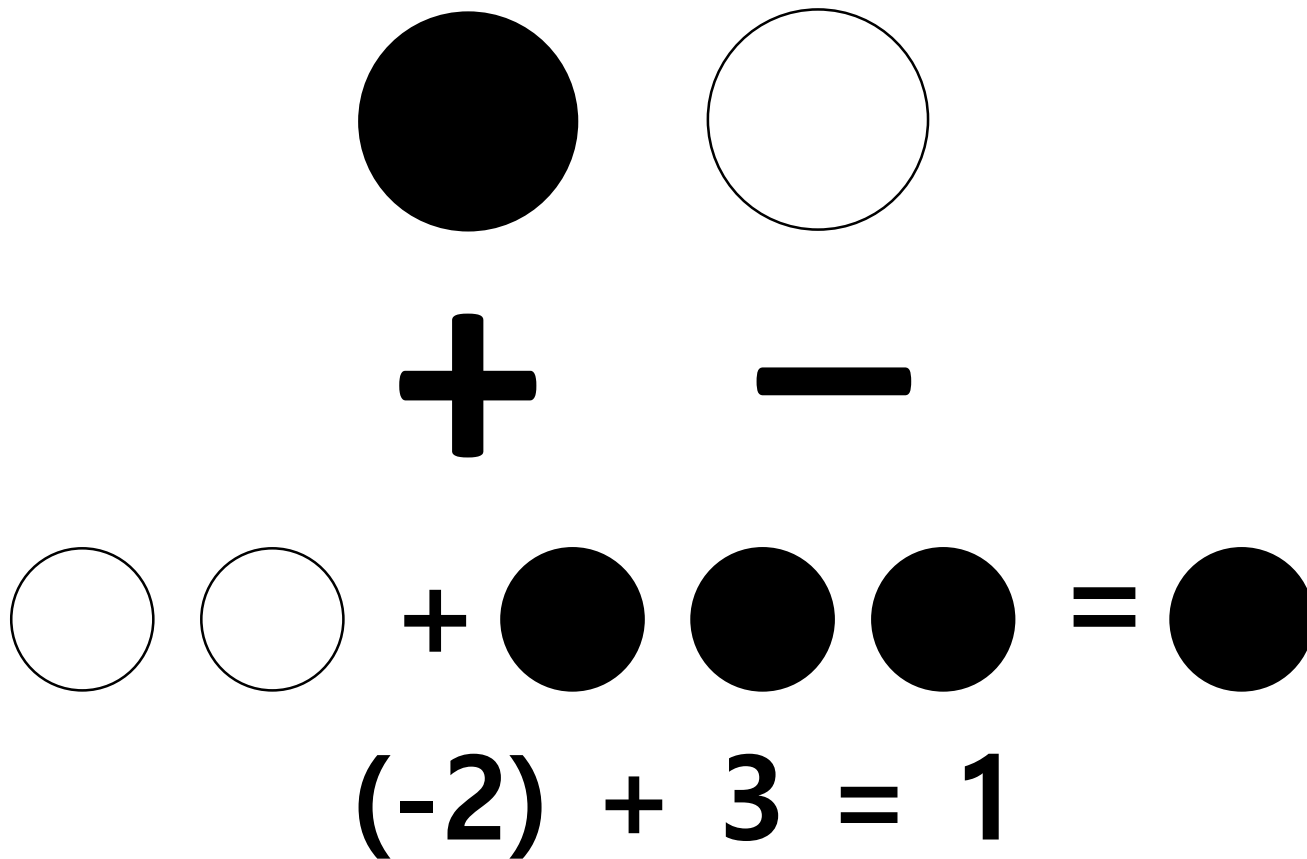


-4 : Symmetrical point of +4 based on 0

학생 활동(수직선 모델)
수직선 모델을 활용한 뱀셈

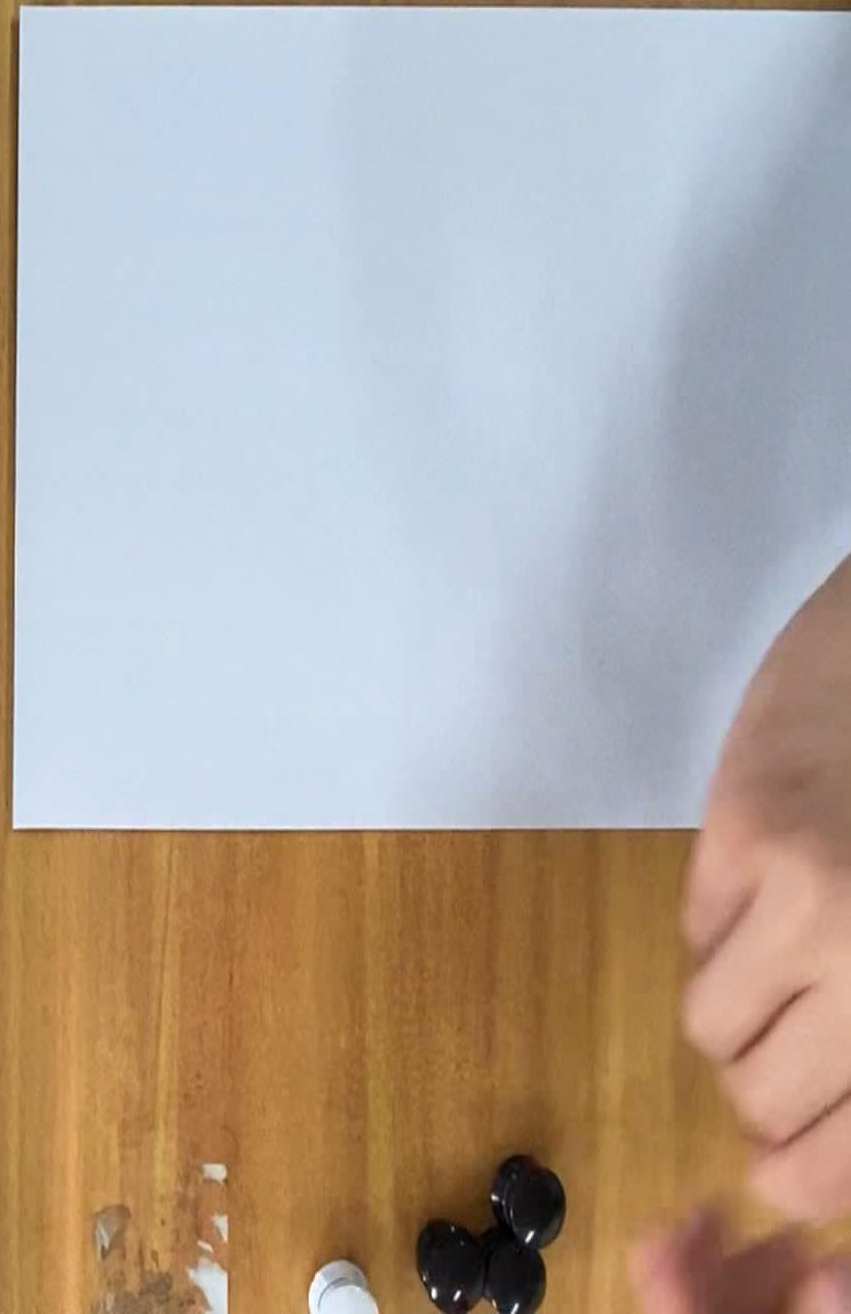


2. Counting Model



$(-2) + 3 = 1$

셈돌 모델의 연산: $(-3) + (+2)$



3. Postman Model

Suppose the postman gives or takes a check (+) and a bill (-).

If the postman takes away the check (+) you have, you will incur a loss, and if the postman gives the bill (-) to you, you will be in debt.

Ex) $(-4) - 2 = -6$

The postman brought bills of 4 and takes away checks of 2 you have, then you owe six debts

우체부가 수령자에게
어음과 고지서를 전달합니다

Check

Bill

Play the postman activity

Write the Sentence and Answer

- 1.
- 2.
- 3.
- 4.
- 5.

Check the right answer
Who is a winner?



Negative Numbers

**Negative numbers are not easy to understand
because we have epistemological obstacle
So When teachers teach the negative numbers,
They need to use intuitive models first and then
Make students accept negative numbers
as a solution for linear equation at the beginning**



The background of the slide features several overlapping, translucent, wavy lines that flow from the bottom left towards the top right. These lines are colored in a gradient, starting with shades of purple and blue on the left, transitioning through green and yellow in the middle, and ending with warm tones of orange and red on the right. The overall effect is one of dynamic movement and vibrant energy.

**Thank you
for attention**